

# A New Gap Phenomenon for Proper Holomorphic Mappings from $\mathbf{B}^n$ into $\mathbf{B}^N$

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In this paper (Math. Res. Lett. 13 (2006). No 4, 509-523), the authors established a pseudo-normal form for proper holomorphic mappings between balls in complex spaces with degenerate rank. This then was used to give a complete characterization for all proper holomorphic maps with geometric rank one, which, in particular, includes the following as an immediate application:

Theorem: Any rational holomorphic map from  $B^n$  into  $B^N$  with  $4 \leq n \leq N \leq 3n - 4$  is equivalent to the D'Angelo map

$$F_\theta(z', w) = (z', (\cos \theta)w, (\sin \theta)z_1w, \dots, (\sin \theta)z_{n-1}w, (\sin \theta)w^2, 0'), \quad 0 \leq \theta \leq \pi/2.$$

It is a well-known (but also quite trivial) fact that any non-constant rational CR map from a piece of the sphere  $\partial B^n$  into the sphere  $\partial B^N$  can be extended as a proper rational holomorphic map from  $B^n$  into  $B^N$  ( $N \geq n \geq 2$ ). By using the rationality theorem that the authors established in [HJX05], one sees that the above theorem (and also the main theorem of the paper) holds in the same way for any non-constant  $C^3$ -smooth CR map from a piece of  $\partial B^n$  into  $\partial B^N$ .

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<http://www.mrlonline.org/mrl/0000-000-00/Huang-Ji-Xu2.pdf>.

(The pdf file of the printed journal version can also be downloaded at <http://www.math.uh.edu/~shanyuji/rank1.pdf>).

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